A New Extrinsic Equivalent Circuit of HEMT's Including Noise for Millimeter-Wave Circuit Design

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***Abstract*—In this paper, we propose a reliable extrinsic equivalent circuit of a high-performance high electron-mobility transistor (HEMT) to determine both the $[S]$-parameters and noise parameters in the millimeter-wave range from characterizations performed below 40 GHz. In the case of the conventional equivalent circuits, only three extrinsic elements have to be determined instead of (at least) eight. We show the validity of the proposed extrinsic equivalent circuit by $[S]$-parameters and noise figure measurements up to the W-band (75–110 GHz). The proposed equivalent circuit is reliable and is very well suited for the design of low-noise integrated circuits for millimeter waves.**

I. Introduction

IN VIEW OF the increasing number of applications in the centimeter-wave range, the millimeter-wave range is now largely used. Multipoint video-distribution system (MVDS) (40.5–42.5 GHz), wireless local area networks (LAN's) (60 GHz), and automotive radar (77 GHz) are among the most focused millimeter-wave applications today. In addition, advanced technologies are now available for manufacturing integrated circuits used in this range.

The main challenge is to design this integrated circuit accurately. To this end, a reliable broad-band high electron-mobility transistor (HEMT) model is needed for designing low-noise amplifiers.

There are two conceivable main ways for determining the microwave and noise parameters of HEMT's for designing low-noise circuits in millimeter waves. First, we can use values of $[S]$-parameters and noise parameters measured directly in the millimeter-wave range. However, it is very difficult to measure the four noise parameters of HEMT's accurately in this frequency range using an impedance-tuning noise-measurement system. In addition, these measured $[S]$-parameters and noise parameters cannot be extrapolated directly either as a function of the frequency, gatewidth, or various bias conditions. The second possibility is to use a conventional equivalent circuit [1], [2] (hereafter referred to as the "conventional equivalent circuit") associated with intrinsic noise sources. The intrinsic and extrinsic elements of this equivalent circuit are determined from measurements performed at relatively low frequencies. Several publications [3], [4] have shown that this conventional equivalent circuit remains valid up to the W-band (75–110 GHz). However, the main drawback of this method is that the calculated millimeter-wave $[S]$-parameters and noise parameters are very sensitive to the value of certain intrinsic and extrinsic elements of this conventional equivalent circuit.

To achieve a good agreement between measured and calculated $[S]$-parameters up to 120 GHz, Tasker and Braunstein [3] increase the complexity of the equivalent circuit by adding input, output, and feedback *LC* distributed networks. However, the determination of each element of such a complex circuit at relatively low frequencies is very sensitive to uncertainty of measurements and to the determination of parasitic parameters. On the other hand, the determination of parasitic elements in the millimeter-wave range may be inaccurate.

Concerning noise calculation of field-effect devices, the Pospieszalski T_g/T_d intrinsic model, commonly used with two equivalent noise temperatures [4], requires an accurate knowledge of the value of certain intrinsic elements like R_i or $G_{\rm d}$, whereas it is well known that the determination of R_i is particularly inaccurate.

To avoid these mentioned equivalent-circuit drawbacks, we have developed a new methodology based on an extrinsic HEMT model, including noise sources which can obtain both the $[S]$-parameters and noise parameters up to the W-band.

After describing this extrinsic model and the associated procedure, we present the variations of the elements of this model versus the gatewidth and biasing conditions. Lastly, we compare the calculated $[S]$-parameters and noise parameters with measurements performed up to the millimeter-waves range.

II. Description of New Methodology

The HEMT equivalent circuit used in this paper is shown in Fig. 1. This circuit includes an active two-port inserted between input and output *LC* cells. The active two-port is described by its admittance ($[Y]^E$), or impedance ($[Z]^E$) matrix. The device noise performance is calculated using two

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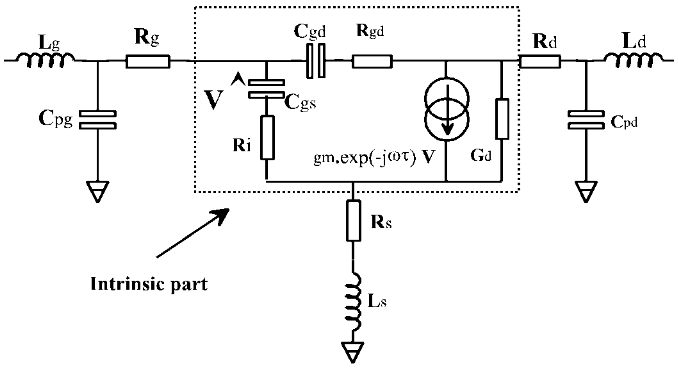


Fig. 1. Conventional small-signal equivalent circuit of the HEMT.

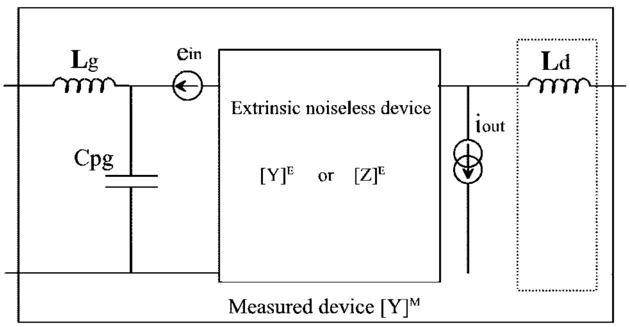


Fig. 2. Proposed extrinsic equivalent circuit of the HEMT, including two noncorrelated noise sources.

uncorrelated noise sources, an input voltage noise source, e_{in}, and output current noise source, i_{out} [5].

The analysis of the intrinsic part of the conventional equivalent circuit of HEMT (see Fig. 1) shows that $[Y]$-parameters exhibit specific variations versus frequency. The imaginary parts of $[Y]$-parameters vary linearly as a function of the frequency, while its real parts vary linearly versus the square of the frequency. In an extension of the work of Minassian [5], we show that these specific variations of $[Y]$-parameters versus frequency remain valid when adding the three access resistance R_{g}, R_{s}, and R_{d} to the intrinsic part of the equivalent circuit, along with the source inductance L_{s} and the gate and drain pad capacitance C_{pg} and C_{pd}. In fact, only the series inductance L_{g} and L_{d} modify these specific variations significantly. The source inductance L_{s} may be a drawback to this method in the millimeter-wave range if it is given a value higher than a few 10^{-11} H. This limitation never occurs in the case of HEMT's designed for on-wafer low-noise applications.

By considering the new equivalent circuit shown in Fig. 2 and the above comments, we can express the $[Y]^E$-parameters by the matrix shown in (1), at the bottom of this page. The matrix (1) remains valid as long as the operating frequency is lower than f_{lim}:

$$f_{\text{lim}} = \frac{1 + R_s(G_m + G_d) + R_d G_d}{2\pi\{C_{\text{gs}}(R_g + R_s) + C_{gd}[R_g + R_d(1 + G_m R_s)]\}} \approx \frac{1}{2\pi\{C_{\text{gs}}(R_g + R_s)\}}. \tag{2}$$

For commonly used devices, f_{lim} is comparable or even greater than the intrinsic cutoff frequency $g_m / \mathbf{2\pi \cdot (C_{gs} + C_{gd})}$. The frequency limitation of the proposed model is then almost the same as the limitation of the conventional equivalent circuit.

Concerning the associated noise model, we define two equivalent noise temperatures T_{in} and T_{out} from the two uncorrelated noise sources e_{in} and i_{out} [5] (see Fig. 2), as follows:

$$T_{\text{in}} = \frac{\overline{e_{\text{in}}{}^2}}{4k\text{Real}\left(\frac{1}{Y_{11}^E}\right)\Delta f} \tag{3}$$

$$T_{\text{out}} = \frac{\overline{i_{\text{out}}{}^2}}{4k\text{Real}\left(\frac{1}{Z_{22}^E}\right)\Delta f} \tag{4}$$

where Z_{22}^E is an element of the following impedance matrix:

$$[Z]_{\text{ext}} = \begin{bmatrix} Z_{11}^E & Z_{12}^E \\ Z_{21}^E & Z_{22}^E \end{bmatrix} = [Y_{\text{ext}}]^{-1}$$
$$\overline{e_{\text{in}} i_{\text{out}}^*} = 0. \tag{5}$$

Compared to conventional intrinsic $T_{\text{g}}/T_{\text{d}}$ two-temperatures models [4], this extrinsic noise model has the following properties.

1) No intrinsic element values are needed to calculate T_{in} and T_{out}.
2) Using a physics-based model [7], it can be shown that the validity of the uncorrelated noise sources is valid through a broad range of drain-current density (up to 200 mA per millimeter of gatewidth).
3) Using the conventional equivalent circuit, we can calculated the extrinsic cross-correlation function $\overline{e_{\text{in}} i_{\text{out}}^*}$ as a function of the intrinsic one $\overline{(e_g i_d^*)}$ as follows:
$$\overline{e_{\text{in}} i_{\text{out}}^*} = (\overline{e_g i_d^*} + R_s \overline{i_d{}^2}). \tag{6}$$
4) Using a physics-based model [7] or experimental results, it can be shown that $\overline{e_g i_d^*}$ presents a low negative real part essentially; at least under a low-noise bias condition ($I_{\text{ds}} < 150$ mA/mm). As a result, the $R_s \overline{i_d{}^2}$ term (positive value) of (6) leads to
$$\overline{e_{\text{in}} i_{\text{out}}^*} \text{ (this work) } \ll \overline{e_g i_d^*} \text{ (Pospieszalski)} . \tag{7}$$
This property of the extrinsic noise model reinforces the uncorrelated noise sources' assumption.

$$[Y]_{\text{ext}} = \begin{bmatrix} Y_{11}^E & Y_{12}^E \\ Y_{21}^E & Y_{22}^E \end{bmatrix} = \begin{bmatrix} (a_{11} \cdot \omega^2 + j \cdot b_{11} \cdot \omega) & (a_{12} \cdot \omega^2 + j \cdot b_{12} \cdot \omega) \\ (a_{21_0} + a_{21} \cdot \omega^2 + j \cdot b_{21} \cdot \omega) & (a_{22_0} + a_{22} \cdot \omega^2 + j \cdot b_{22} \cdot \omega) \end{bmatrix} \tag{1}$$

5) Concerning the cross-correlation function $\overline{e_{\rm in} i^*_{\rm out}}$, we can also easily show that this correlation function increases when the input pad capacitance C_{pg} is added. Consequently this parasitic input capacitance has to be deembedded.

In summarizing the main advantage of this extrinsic model (compared to the intrinsic ones), only three extrinsic elements ($L_{\rm g}$, C_{pg}, and $L_{\rm d}$) have to be determined instead of eight (at least). As compared to the conventional equivalent circuit, this model is less sensitive to measurements uncertainty and more reliable for millimeter-wave integrated-circuit design. However, due to the lack of information concerning the intrinsic elements and certain access elements, this method is not suited to HEMT physical studies or to the optimization of fabrication processes.

To determine the elements of the model, we first determine the coefficients a_{ij} and b_{ij} of the matrix (1) from the measured $[S]$-parameters after deembedding the three extrinsic elements $L_{\rm g}$, C_{pg}, and $L_{\rm d}$. These extrinsic elements are obtained using a conventional "cold FET" method [1] or the new approach described in [8]. These coefficients are calculated by simple linear regression of $[Y]^E$ versus either frequency or the square of the frequency. We then obtain the two equivalent noise temperatures from noise-figure measurements [9].

The $[S]$-parameters and noise figure of the HEMT are measured in the 5–40-GHz frequency range using a conventional on-wafer probe station. These relatively low operating frequencies greatly simplified the measurement procedure. As compared with measurements facilities in the V- or W-bands (the measurement tools and its associated calibration procedure) provides accurate and reliable measurements in this frequency range (5–40 GHz).

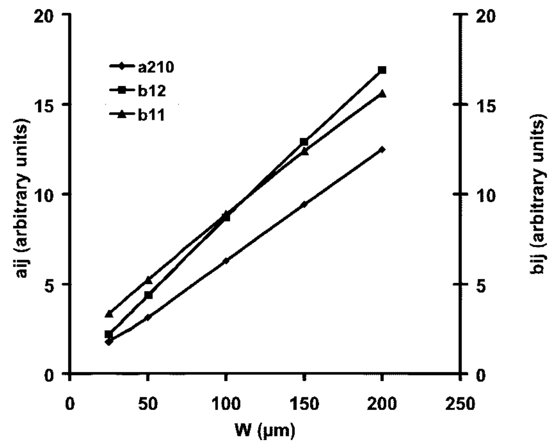
The two previously mentioned steps are performed for devices with different gatewidths and for different biasing conditions. For a given drain to source voltage ($V_{\rm ds}$), the variations of the elements (a_{ij}, b_{ij}, $T_{\rm in}$, and $T_{\rm out}$) of this model versus gatewidth and drain-current density, can be fitted by simple first- or second-order polynomial functions. This aspect of the proposed model is very useful to simplify the design process of integrated circuits.

III. Experimental Results

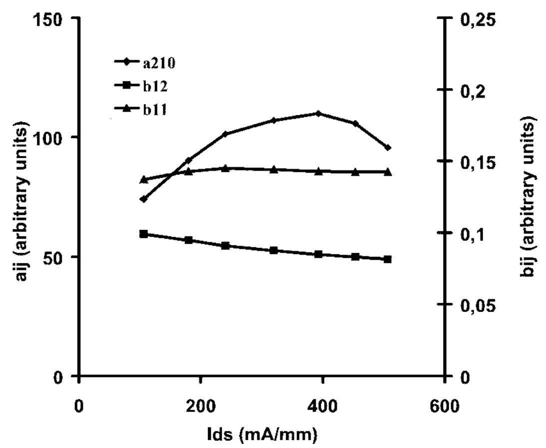
In this section, we present the experimental data of this model for a pseudomorphic HEMT (pHEMT) with a 0.1-μm gate length.

In Fig. 3(a), we show the variations of some coefficients a_{ij}, b_{ij} (1) deduced from the measured $[S]$-parameters versus gatewidth for a given drain current ($I_{\rm ds} = 100$ mA/mm; $V_{\rm ds} = 3$ V). As shown in Fig. 3(a), the variations of the coefficients a_{ij}, b_{ij} versus gatewidth can be fitted using very simple polynomial functions. In fact, these coefficients show almost the same scaling rule of the intrinsic elements of the conventional equivalent circuit.

The variations of these coefficients versus drain-current density for a given gatewidth (150 μm) are represented in Fig. 3(b). It should be noted that the variations of the coefficients a_{ij}, b_{ij}, as a function of gatewidth and drain current,



(a)



(b)

Fig. 3. (a) Variations (versus the gatewidth) of the a_{ij} and b_{ij} coefficient of the $[Y]^E$ matrix of the proposed equivalent circuit in the case of a 0.1 -μm gate-length pHEMT. $I_{\rm ds} = 100$ mA/mm, $V_{\rm ds} = 3$ V. (b) Variations (versus the drain-current density) of the a_{ij} and b_{ij} coefficient of the $[Y]^E$ matrix of the proposed equivalent circuit in the case of a 0.1-μm gate-length pHEMT. $W = 150$ μm, $V_{\rm ds} = 3$ V.

are quite similar than the variations of the intrinsic elements of the conventional equivalent circuit.

Fig. 4(a) shows the variation of the two equivalent noise temperatures $T_{\rm in}$ and $T_{\rm out}$ as a function of gatewidth, where the drain-current density is 100 mA/mm ($V_{\rm ds} = 3$ V). These temperatures are almost independent of the gatewidth. Moreover, Fig. 4(b) shows that $T_{\rm out}$ is directly related to the drain current while $T_{\rm in}$ is constant and close to the ambient temperature.

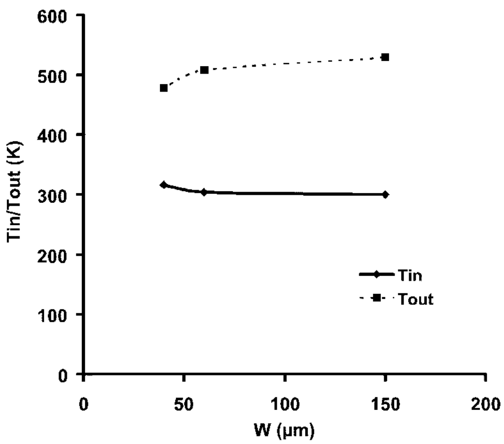
To summarize, using simple fitting functions (first- or second-order polynomial functions), the variations of the parameters a_{ij}, b_{ij}, $T_{\rm in}$, and $T_{\rm out}$ versus gatewidth and drain current (for a given $V_{\rm ds}$) constitute the inputs data for the design of millimeter-wave integrated circuits.

IV. Validation of the Model in Millimeter-Wave Frequency Range

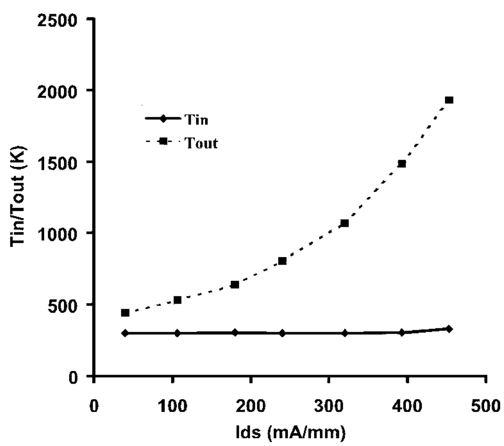
$[S]$-parameters and noise parameters in the millimeter-wave range of an HEMT can obviously be calculated from the

TABLE I
ERRORS BETWEEN CALCULATED AND MEASURED $[S]$ PARAMETERS (IN PERCENTAGE (%) FOR MAGNITUDE AND IN DEGREES FOR PHASE)

Frequency Range	$\Delta\lvert S_{11}\rvert$	$\Delta\lfloor S_{11}$	$\Delta\lvert S_{12}\rvert$	$\Delta\lfloor S_{12}$	$\Delta\lvert S_{21}\rvert$	$\Delta\lfloor S_{21}$	$\Delta\lvert S_{22}\rvert$	$\Delta\lfloor S_{22}$
1–50 GHz	<2%	±2°	<5%	±2°	<1%	±1°	<2%	±7°
50–75 GHz	<6%	±2°	<6%	±6°	<6%	±6°	<5%	±10°
75–110 GHz	<9%	±3°	<10%	±10°	<9%	±10°	<7%	±10°

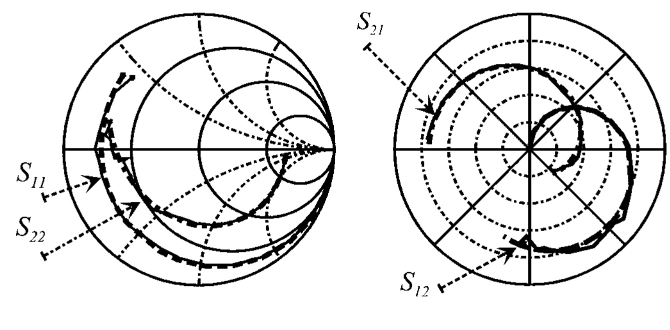


(a)



(b)

Fig. 4. (a) Variations (versus the gatewidth) of the $T_{\rm in}$ and $T_{\rm out}$ equivalent noise temperatures of the proposed equivalent circuit in the case of a 0.1-μm gate-length pHEMT. $I_{\rm ds} = 100$ mA/mm, $V_{\rm ds} = 3$ V. (b) Variations (versus the drain-current density) of the $T_{\rm in}$ and $T_{\rm out}$ equivalent noise temperatures of the proposed equivalent circuit in the case of a 0.1-μm gate-length pHEMT. $W = 150$ μm, $V_{\rm ds} = 3$ V.



(a) (b)

Fig. 5. Comparison of measured and calculated scattering parameters in the 1–110-GHz frequency range. The device is a pHEMT of 0.15 × 2 × 25 μm^2; $V_{\rm ds} = 2$ V, $V_{\rm gs} = 0$ V. (a) Reflection coefficients. (b) Transmission coefficients, dashed lines correspond to extrapolated parameters, full lines correspond to measured parameters. The modulus of S_{12} is multiplied by ten as a matter of clarity of the graph.

proposed extrinsic model. To show the validity of this model, we have to compare the calculated and measured data down to millimeter waves.

Concerning the measurements of $[S]$-parameters in the V-band (50–75 GHz) and W-band (75–110 GHz), we use a millimeter-wave automatic-network analyzer (MWANA) and Picoprobe V- and W-bands probes. These probes are connected to the MWANA with rectangular WR15 or WR10 waveguides. The probes were chosen and the waveguide connections were optimized to reduce the losses and increase the directivity of the bench. This point is especially important for the noise measurements.

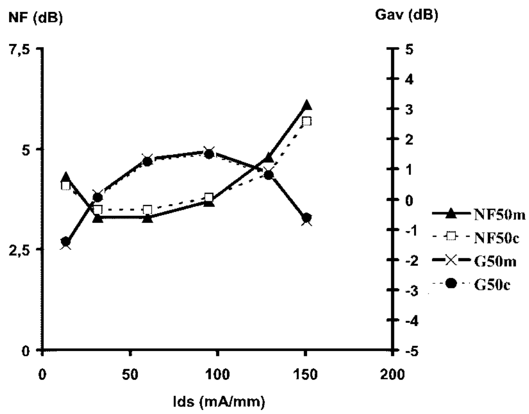
Fig. 5(a) and (b) shows the good agreement between the measured $[S]$-parameters and calculated ones using the proposed model in the 1–110-GHz frequency range. The device is a 0.15 × 2 ×25 μm^2 pHEMT; $V_{\rm ds} = 2$ V, $V_{\rm gs} = 0$ V. The calculated $[S]$-parameters are deduced from the parameters a_{ij}, b_{ij} and from the extrinsic elements $L_{\rm g}$, $L_{\rm d}$. These parameters are determined from 5–40-GHz measurements following the procedure described in Section III.

The errors between calculated and measured $[S]$-parameters are shown in Table I.

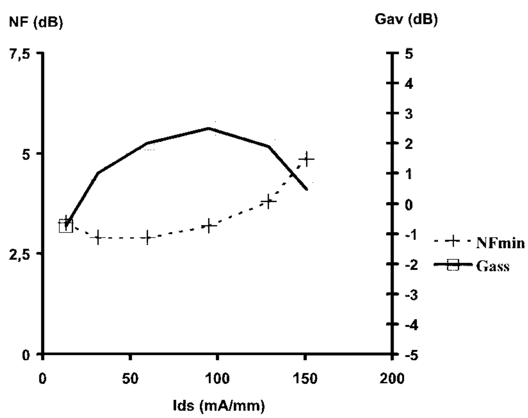
For the noise performance, the proposed model is validated by comparing the noise figure with a 50-Ω generator impedance ($\rm NF_{50}$) in the measurement process and in calculation model. $\rm NF_{50}$ is a good criterion to show the accuracy obtained by the noise model in the four noise parameters because $\rm NF_{50}$ precisely depends on the four noise parameters of the device.

To show the validity of the calculated noise parameters using the extrinsic equivalent circuit, we have developed a narrow-band on-wafer noise measurement setup at 60 and 94 GHz. Using such benches, we are able to measure $\rm NF_{50}$ with an estimated accuracy of ±0.3 dB, which is quite reasonable, considering these very high frequencies.

Fig. 6(a) shows the good agreement of the measured and calculated $\rm NF_{50}$ and the available associated gain (G_{av50}) versus the drain current in the case of a 2 × 50 × 0.1 μm^2 LM-HEMT at 94 GHz. Fig. 6(b) shows the variation of the calculated minimum noise figure ($\rm NF_{min}$) and available associated gain ($G_{\rm ass}$) versus the drain current. $\rm NF_{min}$ and



(a)



(b)

Fig. 6. (a) Comparison between measured and calculated minimum noise figure and available gain with a 50-Ω source impedance (NF_{50} and G_{av50}) as a function of I_{ds} at 94 GHz. The device is a $2 \times 50 \times 0.1\,\mu\mathrm{m}^2$ LM-HEMT; $V_{\mathrm{ds}} = 0.8$ V. (b) Calculated minimum noise figure ($\mathrm{NF}_{\min}$) and available associated gain (G_{ass}) as a function of I_{ds} at 94 GHz. The device is a $2 \times 50 \times 0.1\ \mu\mathrm{m}^2$ LM-HEMT; $V_{\mathrm{ds}} = 0.8$ V.

G_{ass} are calculated using the parameters a_{ij}, b_{ij}, T_{in}, and T_{out} of the proposed model obtained in lower frequency range.

V. Conclusion

An extrinsic equivalent circuit of an HEMT including noise sources is presented. Compared with a conventional circuit, the main advantage of this model is that only three extrinsic elements have to be determined instead of eight or more, as is true of the conventional equivalent circuit. Moreover, each parameter of this proposed equivalent circuit can be accurately deduced from the S-parameter and noise measurements performed in relatively low operating frequencies (up to 40 GHz).

This proposed model can be introduced easily in commercially available microwave simulators to design millimeter-waves low-noise amplifiers. Finally, we have shown the validity of this model by comparison between calculated and measured $[S]$-parameters and noise figures up to the W-band.

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